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OF SANDWICH STRUCTURES
AT HIGH TEMPERATURE CONDITIONS

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ANALYSIS OF THERMAL CHARACTERISTICS OF SANDWICH
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The samples of three-layer sandwich panels in the form of either equal-cells thin-walled honeycomb cores, corrugated cores or a set of structure stiffeners are shown in fig. 1. Such panels, possessing high heatshielding and strength properties, may be widely used in the structures, applied at high temperatures, and also in combination with other heatshielding, load-carrying and isolating layers. The analysis of temperature fields and heat transfer of the multilayer sandwich structures reduces to a general scheme when regarding the layer of the core as a structurally-anisotropic one with effective thermal characteristics: heat capacity, heat transfer in different directions and emissivity. The first of them is an averaged heat capacity of the elements of the equal cells of the core and the rest ones are determined either through the analysis of complex heat and radiation conduction within the cell or experimentally.

Let us consider temperature field and radiation conduction within the shown in fig. 1a cylindrical cell of arbitrary plan form, heat shielded along the side face and with a specified temperatures of the top and bottom bases, T_1 and T_2 .

L is the length of the bases of inconcave contour, F is the base area, δ , λ , ϵ are the thickness, heat conduction and emissivity of the wall, respectively, and H is the

height. We shall average the radiation fluxes over the every section parallel to bases, considering the problem as a onedimensional, which corresponds exactly to the reality when the core is of a circular cylindrical form or has the form of a set of parallel walls (two-dimensional cases).

The equation of the steady-state heat transfer within the wall of the cell is:

$$\frac{d^2 t(\xi)}{d\xi^2} - \sigma q(\xi) = 0, \quad 0 < \xi < 1, \quad (1)$$

the integral equation of the radiation conduction is

$$\begin{aligned} \frac{q(\xi)}{\varepsilon} = & t^4(\xi) - \left[t_1^4 - \frac{1-\varepsilon_1}{\varepsilon_1} q_1 \right] K_0(\xi) - \left[t_2^4 - \frac{1-\varepsilon_2}{\varepsilon_2} q_2 \right] K_0(1-\xi) - \\ & - \int_0^\xi \left[t^4(\eta) - \frac{1-\varepsilon}{\varepsilon} q(\eta) \right] K(\xi-\eta) d\eta - \int_\xi^1 \left[t^4(\eta) - \frac{1-\varepsilon}{\varepsilon} q(\eta) \right] K(\eta-\xi) d\eta, \end{aligned} \quad (2)$$

the boundary conditions are:

$$t(0) = t_1, \quad t(1) = t_2, \quad (3)$$

where

$$t = \frac{T}{T_n}, \quad \xi = \frac{x}{H}, \quad q = \frac{q_{res}}{C_0 T_n^4}, \quad \sigma = \frac{C_0 T_n^3 H^2}{\delta \lambda};$$

$K_0(\xi)$, $K_0(1-\xi)$ are the average coefficients of the cell wall elements irradiancy by the top and bottom bases, respectively;

$K(|\xi-\eta|) = -\frac{dK_0(|\xi-\eta|)}{d|\xi-\eta|}$ is a coefficient of a mutual irradiancy of the wall elements, which are at a distance of $|\xi-\eta|$ from each other; q_1 , q_2 are the averaged densities of the resul-

ting radiation from the top and bottom bases, determined from the equations:

$$\frac{q_1}{\varepsilon_1} = t_1^4 - \int_0^1 \left[t^4(\eta) - \frac{1-\varepsilon}{\varepsilon} q(\eta) \right] K_0(\eta) \frac{LH}{F} d\eta - \int_1^\infty \left[t_2^4 - \frac{1-\varepsilon_2}{\varepsilon_2} q_2 \right] K_0(\eta) \frac{LH}{F} d\eta, \quad (4)$$

$$\frac{q_2}{\varepsilon_2} = t_2^4 - \int_0^1 \left[t^4(\eta) - \frac{1-\varepsilon}{\varepsilon} q(\eta) \right] K_0(1-\eta) \frac{LH}{F} d\eta - \int_{-\infty}^0 \left[t_1^4 - \frac{1-\varepsilon_1}{\varepsilon_1} q_1 \right] K_0(1-\eta) \frac{LH}{F} d\eta.$$

For the arbitrary form of the bases we shall approximate the function $K_0(\xi)$, depending upon the cell form and rapidly decaying with the increase of ξ , by the exponential relationship:

$$K_0(\xi) = \frac{1}{2} e^{-2f\xi} = \int_{-\infty}^0 K(1\xi - \eta) d\eta, \quad (5)$$

where $f = \frac{LH}{4F}$ is a nondimensional geometric parameter. Fig. 2 gives the comparison of the accurate irradiancy coefficients and their first derivatives with those of computed from the equation (5) for the circular cylindrical core ($f = H/D$ [1]) and for the core in the form of a set of parallel walls ($f = H/2D$).

Using the equation (5) the integral equation (2) may be reduced to the differential form:

$$\frac{d^2 q(\xi)}{d\xi^2} - 4\varepsilon f^2 q(\xi) = \varepsilon \frac{d^2 t^4(\xi)}{d\xi^2} \quad (6)$$

with the boundary conditions:

$$\frac{q(0)}{\varepsilon} = t_1^4 - \frac{1}{2} \left(t_2^4 - \frac{1-\varepsilon_2}{\varepsilon_2} q_2 \right) - \frac{e^{-2f}}{2} \left(t_2^4 - \frac{1-\varepsilon_2}{\varepsilon_2} q_2 \right) - f \int_0^1 \left[t^4(\eta) - \frac{1-\varepsilon}{\varepsilon} q(\eta) \right] e^{-2f\eta} d\eta, \quad (7)$$

$$\frac{q(1)}{\varepsilon} = t_2^4 - \frac{e^{-2f}}{2} \left(t_1^4 - \frac{1-\varepsilon_1}{\varepsilon_1} q_1 \right) - \frac{1}{2} \left(t_2^4 - \frac{1-\varepsilon_2}{\varepsilon_2} q_2 \right) - f \int_0^1 \left[t^4(\eta) - \frac{1-\varepsilon}{\varepsilon} q(\eta) \right] e^{-2f(1-\eta)} d\eta,$$

and the relations (4) change to the form:

$$\begin{aligned} \frac{q_1}{\varepsilon_1} &= t_1^4 - 2f \int_0^1 [t^4(\eta) - \frac{1-\varepsilon}{\varepsilon} q(\eta)] e^{-2f\eta} d\eta - (t_2^4 - \frac{1-\varepsilon_2}{\varepsilon_2} q_2) e^{-2f}, \\ \frac{q_2}{\varepsilon_2} &= t_2^4 - 2f \int_0^1 [t^4(\eta) - \frac{1-\varepsilon}{\varepsilon} q(\eta)] e^{-2f(1-\eta)} d\eta - (t_1^4 - \frac{1-\varepsilon_1}{\varepsilon_1} q_1) e^{-2f}. \end{aligned} \quad (8)$$

So the problem is reduced to two differential equations of the second order (1) and (6) with the boundary conditions (3) and (7) and with the relations (8). Their linearization with respect to $t(\xi) - 1$ permits this solution to be obtained in the closed form:

$$\begin{aligned} t(\xi) &= t_2 - (t_1 - t_2)\xi + c_1 \frac{\sigma [e^{k\xi} - 1 + (1 - e^k)\xi]}{4\varepsilon(\sigma + f^2)} + c_2 \frac{\sigma [e^{k(1-\xi)} - e^k - (1 - e^k)\xi]}{4\varepsilon(\sigma + f^2)}, \\ q(\xi) &= c_1 e^{k\xi} + c_2 e^{k(1-\xi)}, \end{aligned} \quad (9)$$

where

$$c_i = (-1)^i \frac{2\varepsilon(2-\varepsilon_i)(\sigma + f^2)(t_1 - t_2)}{f[A_i - (-1)^i(\varepsilon_1 - \varepsilon_2)B_i]}.$$

Here

$$A_i = (2 - \varepsilon_i) \left[\left(\frac{kf}{2} + \frac{\sigma}{f} \right) e^k - \frac{\sigma}{f} + \frac{kf}{2} \right] - \varepsilon_i (\sigma + f^2) (1 - e^k),$$

$$B_i = k \frac{1 - e^k}{2} \frac{(2 - \varepsilon_i) \left[\frac{\sigma}{f} - e^k \left(\frac{\sigma}{f} + kf \right) \right] - 2\varepsilon_i e^k (\sigma + f^2)}{(2 - \varepsilon_i) \left[f\varepsilon(2 - \varepsilon_j) - \frac{k\varepsilon_j}{2} \right] - (2 - \varepsilon_j) e^k \left[f\varepsilon(2 - \varepsilon_i) + \frac{k\varepsilon_i}{2} \right]},$$

$$i = 1, 2, \quad j = 3 - i,$$

$$k = 2\sqrt{\varepsilon(\sigma + f^2)}.$$

We determine the effective heat transfer of the core in the direction of the Z-axis of the cell as follows:

$$\lambda_{eff} = \frac{QH}{F(T_1 - T_2)}, \quad (10)$$

where Q is the heat flux, flowing from the bottom base to the upper one.

Using the solution (9), we shall obtain the expression for λ_{eff} in the nondimensional form:

$$\frac{\lambda_{eff}H}{\lambda\delta} = 4 \left\{ f + \frac{\sigma}{2f} \frac{f(\kappa+1-e^\kappa)(2-\varepsilon_1) - 2\varepsilon_1(\sigma+f^2)}{A_1 + (\varepsilon_1 - \varepsilon_2)B_1} + \right. \\ \left. + \frac{\sigma}{2f} \frac{f(\kappa e^\kappa + 1 - e^\kappa)(2-\varepsilon_1) + 2\varepsilon_1(\sigma+f^2)e^\kappa}{A_2 + (\varepsilon_2 - \varepsilon_1)B_2} \frac{2-\varepsilon_2}{2-\varepsilon_1} \right\}, \quad (11)$$

depending on the temperature σ and on the characteristics of the cell δ/H , f , ε , ε_1 , ε_2 .

With small effect of radiation ($\sigma \ll 1$)

$$\lambda_{eff} = \lambda \frac{\delta L}{F} \quad (12)$$

and with small effect of heat conduction ($\sigma \gg 1$)

$$\lambda_{eff} = 4\lambda \frac{\delta}{H} \sigma \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2 (1-f)} = 4C_0 T_n^3 H \varepsilon_{12} \quad (13)$$

The expression (13) at $\varepsilon_1 = \varepsilon_2 = 1$ corresponds to an approximate solution of Vlasov-Hottel's problem /ref. 2/ of reradiation within the cylindrical duct, well agreed with the numerical solution /ref. 1 and 2/, which confirms the possibility of used by us exponential approximation (expression 5) of the configuration factor.

When the emissivity of the cell wall $\varepsilon = 0$ the effective

heat transfer is determined by the sum of the expressions (12) and (13)

$$\lambda_{eff} = 4\lambda \frac{\delta}{H} \left[f + \sigma \frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2 (1-f)} \right] = \lambda \frac{\delta L}{F} + 4C_0 T_n^3 H^2 \epsilon_{12} \quad (14)$$

The results of computation of λ_{eff} from the formulae (11) and (14) at $\epsilon_1 = \epsilon_2 = \epsilon = 0, \delta$ are shown in fig. 3 by the solid and dotted curves. At $\epsilon_1, \epsilon_2 > 0, \delta$ it appears possible to compute independently the heat flux by radiation and heat conduction and approximate effective heat transfer from the elemental formula (14). This important circumstance permits the obtained solution to be used for the computation of the effective heat transfer of the core in various directions when the wall of the cell is of nonuniform thickness (honeycombs, fig. 1a) and when the cell has two cavities (fig. 1d) and so on. At certain conditions there exists a form of the core cells, with which the minimal effective heat transfer occurs at a specified temperature. In the case of $\epsilon = 0$ from the equation (14) we shall obtain, for the optimal geometrical parameter f , the expression:

$$f = \sqrt{\sigma} - \frac{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2}{\epsilon_1 \epsilon_2} \quad \text{at } \sqrt{\sigma} \geq \frac{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2}{\epsilon_1 \epsilon_2} \quad (15)$$

With the absence of one of the skins for the structures shown in fig. 1 the conception of the effective emissivity and absorption capability is introduced. For this let us consider the temperature field and radiation conduction of the cylindrical cell (see fig. 1a) at a given averaged density of the radiation flow

$q_{\text{incident}} = C_0 T_n^4 \bar{q}_{\text{incident}}$, incident on the cell from above and at a given temperature of the bottom base T_1 . The set of

equations, similar to (1), (2) and (4) for a given case, is:

$$\frac{d^2 t(\xi)}{d\xi^2} - \sigma q(\xi) = 0, \quad 0 < \xi < 1,$$

$$\frac{q(\xi)}{\varepsilon} = t''(\xi) - \left(t_1'' - \frac{1-\varepsilon_1}{\varepsilon_1} q_1\right) K_0(\xi) - \bar{q}_{inc} K_0(1-\xi) -$$

$$- \int_0^\xi \left[t''(\eta) - \frac{1-\varepsilon}{\varepsilon} q(\eta)\right] K(\xi-\eta) d\eta - \int_\xi^1 \left[t''(\eta) - \frac{1-\varepsilon}{\varepsilon} q(\eta)\right] K(\eta-\xi) d\eta, \quad (16)$$

$$\frac{q_1}{\varepsilon_1} = t_1'' - \int_0^1 \left[t''(\eta) - \frac{1-\varepsilon}{\varepsilon} q(\eta)\right] K_0(\eta) \frac{LH}{F} d\eta - \int_1^\infty \bar{q}_{inc} K_0(\eta) \frac{LH}{F} d\eta,$$

the boundary equations are:

$$t(0) = t_1, \quad \frac{dt(1)}{d\xi} = 0. \quad (17)$$

We shall obtain an approximate solution of the linearized set (16), (17) in the form:

$$t(\xi) = t_1 + C_1 \frac{\sigma (Sh k \xi - \xi K Ch k)}{4\varepsilon(\sigma + f^2)} + C_2 \frac{\sigma [Ch k (1-\xi) - Ck k]}{4\varepsilon(\sigma + f^2)}, \quad (18)$$

$$q(\xi) = C_1 Sh k \xi + C_2 Ch k (1-\xi),$$

where

$$C_1 = k \left[1 + 4(t_1 - 1) - \bar{q}_{inc} \right] \frac{f Sh k + \frac{2\varepsilon_1(f^2 + \sigma)}{(2-\varepsilon_1)k} Ch k}{\varphi(\sigma, f, \varepsilon_1, \varepsilon)},$$

$$C_2 = k \left[1 + 4(t_1 - 1) - \bar{q}_{inc} \right] \frac{f + \frac{\sigma}{f} Ch k}{\varphi(\sigma, f, \varepsilon_1, \varepsilon)}.$$

Here:

$$\varphi(\sigma, f, \varepsilon_1, \varepsilon) = \frac{4f}{k} \left(f + \frac{\sigma}{f} Ch k \right)^2 +$$

$$+ \left[f Sh k + \frac{2\varepsilon_1(\sigma + f^2)}{k(2-\varepsilon_1)} Ch k \right] \left[(4\sigma + \frac{2\sigma}{f} + 2f) Ch k + \frac{4f^2}{k} Sh k \right].$$

Estimating the density of the effective heat flow, radiated by the cavity of the cell through the upper open base, we shall obtain:

$$q_{eff} = \varepsilon_{eff} [1 + 4(t_2 - 1)] C_0 T_n^4 + (1 + \varepsilon_{eff}) q_{inc} \approx \varepsilon_{eff} C_0 T_1^4 + (1 - \varepsilon_{eff}) q_{inc}, \quad (19)$$

where

$$\varepsilon_{eff} = \frac{[fShk + \frac{2\varepsilon_1(\sigma + f^2)}{(2 - \varepsilon_1)k} Chk] 4(f + \frac{\sigma}{f}) Chk}{\varphi(\sigma, f, \varepsilon_1, \varepsilon)} \quad (20)$$

Thus the surface of the stiffened structure, consisted of the open equal cells, may be regarded as radiative and absorptive one in the integral sense similarly to the grey surface with the temperature T_1 and with the effective emissivity ε_{eff} , depending on the temperature σ and characteristics of the cell $f, \varepsilon_1, \varepsilon_2$ (see fig. 4).

In the particular case when $\sigma = 0, \varepsilon = \varepsilon_1$ the solution (20) permits the effective emissivity of the isothermal cavities /ref. 4/ to be obtained. The comparison of the values ε_{eff} , obtained numerically in the ref. 4 for the circular cylinder with different values of ratio $f = H/D$, with those of computed from the formula (20) at $\sigma = 0, \varepsilon = \varepsilon_1 = 0,5$ is given in the table:

f	0,25	0,5	1,0	2,0	3,0	4,0
ε_{eff} from ref./4/	0,6569	0,7424	0,8024	0,8331	0,8359	0,8367
ε_{eff} From the formula (20)	0,6616	0,7449	0,8079	0,8272	0,8284	0,8284

For the general case of the corrugated core the problem of determination of the effective thermal characteristics of \mathcal{R}_{eff} and ε_{eff} reduces to the solution of the nonlinear integro-differential equations of one-dimensional steady-state heat and radiation conductions expressed as (1) - (3), (16), (17), describing the heat transfer within the adjacent cavities of the core cell, shown in fig. 1b. The problem has been solved in finite differences by the timeasymptotic method. The design equations for the determination the temperatures and the radiation fluxes elements of the cell contour at the boundary conditions of (3) are as follows (for one of the cavities):

$$t_{m+1}^{j+1} = t_{m+1}^j (1-3Fo) + Fo(t_{m+2}^j + 2t_1) - \frac{\sigma Fo}{2n^2 \sin^2 \alpha} (q_{1,m+1}^j + q_{2,m+1}^j),$$

$$t_i^{j+1} = t_i^j (1-2Fo) + Fo(t_{i-1}^j + t_{i+1}^j) - \frac{\sigma Fo}{2n^2 \sin^2 \alpha} (q_{1,i}^j + q_{2,i}^j),$$

$$i = m+2, \dots, m+n-1, \quad (21)$$

$$t_{m+n}^{j+1} = t_{m+n}^j (1-3Fo) + Fo(2t_2 + t_{m+n-1}^j) - \frac{\sigma Fo}{2n^2 \sin^2 \alpha} (q_{1,m+n}^j + q_{2,m+n}^j),$$

$$t_i^j = t_1, \quad i = 1, 2, \dots, m, \quad t_i^j = t_2, \quad i = m+n+1, \dots, N,$$

$$t_i^0 = t_1 - (t_1 - t_2) \frac{2(i-m)-1}{2n}, \quad i = m+1, \dots, m+n,$$

$$\frac{q_{2,i}^j}{\varepsilon_i} = t_i^{j4} - \sum_{k=1}^N \left(t_k^{j4} - \frac{1-\varepsilon_k}{\varepsilon_k} q_{1,k}^j \right) [\varphi(i,k) + \varphi(i, 2N+1-k)], \quad (22)$$

where

Fo is a nondimensional step of computation with time;

$\varphi(i,k)$ are configuration factor, determined by the method of stretched treads;

$n, 2m, 2N$ are the number of elements, by which the wall and one of the bases and the cell contour are divided respectively.

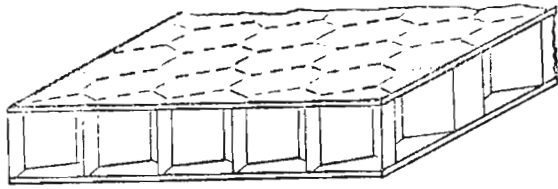
After coming to the steady-state condition the effective heat conduction of the corrugated core is being computed by the formula:

$$\frac{\lambda_{eff} H}{\lambda \delta} = \frac{1}{(N-n)(t_1-t_2)} \left[4n^2 \sin^2 \alpha (t_1-t_{m+1}) + 6 \left(\sum_{i=1}^m q_{1,i} + \sum q_{2,i} \right) \right]. \quad (23)$$

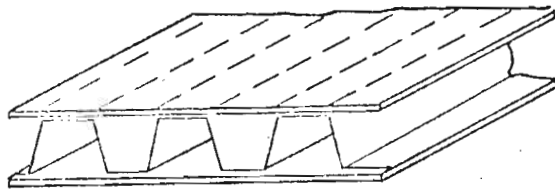
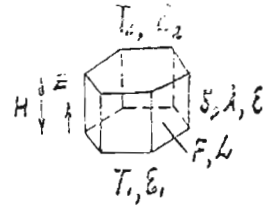
The results of the numerical solution, which are in a good agreement with the computation by the formula (11), are shown by the dash-and-dot lines in fig. 3 for the particular case of an orthogonal corrugation.

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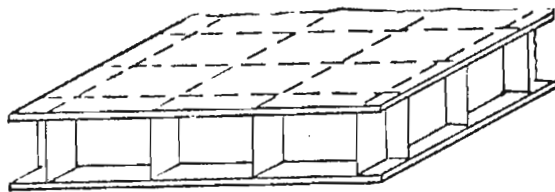
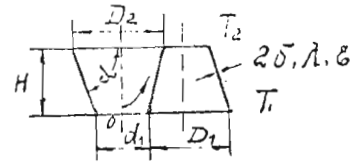
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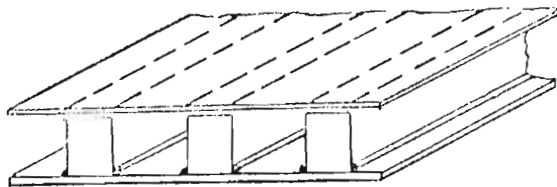
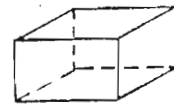
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